MULTIPLE DIPOLE ATTRACTIONS IN MIRROR PLATES 11/09/11

Norman Albers <nvalbers@gmail.com>

In a recent paper I detailed the attractive force between a dipole element on the surface of a metallic reflecting plate, and its induced opposite on a nearby plate.[1] Now we examine the multiplier effect of dipoles laterally offset from the first, directly opposing pair of charges. Offsets of not 'many' lattice dimensions 'L' will have the same interaction and raise the magnitude of these Vander Waals forces. Consider the expansion process by which the fourth-order term is derived.

Plate separation is 'a' and the image charge is as if it were at '$2a$'. Thus the initial solution for the directly opposing pair; we start by considering either charge, as each is a bit nearer its attracting opposite, than to its repelling similar charge. Figuring the slightly diagonal distances, the force will be:

$$F_1 = \frac{e^2}{4\pi\epsilon_0}\left\{\frac{1}{(2a)^2} - \frac{\cos\alpha}{(2a)^2+(L/2)^2}\right\} \quad .$$

The angle α has a tangent of $\frac{L/2}{2a}$. Let us for now leave off the coefficient fraction on the left. We see that:

$$F_1 = \frac{1}{(2a)^2}\{1 - [1+(L/4a)^2]^{-3/2}\} \quad .$$

Since we assume L/a is small compared to 1, expand to read:

$$F_1 = \frac{1}{(2a)^2}\{3/2\,(L/4a)^2\} \;=\; 3/128\,L^2/a^4 \quad .$$

The other charge in the dipole experiences the same attraction, so the two contributions are added to say: $F_I = 3/64\,L^2/a^4$. This was the conclusion of the first paper. Let us now consider mutual attraction between two dipoles a few lattice lengths offset laterally. Since the symmetry is not so clear, let us add the attraction and repulsion for the +/- pair. Arbitrarily we analyze for an offset of two lattice cells, say the dipole on the left plate is two atoms higher than the one on the right. The upper charge of the RH pair we call *P,* and the lower opposite charge, Q. It is clear there is net repulsion on *P* but attraction on Q:

$$F_{III\,P} = -\frac{\cos\alpha}{(2a)^2+(3/2\,L)^2} + \frac{\cos\beta}{(2a)^2+(2L)^2} \;, \qquad F_{IIIQ} = \frac{\cos\alpha}{(2a)^2+(2L)^2} + \frac{\cos\beta}{(2a)^2+(5/2L)^2} \;.$$

As before the cosines combine easily to give:

$$F_{III\,P} = -[(2a)^2+(3/2\,L)^2]^{-3/2} + [(2a)^2+(2L)^2]^{-3/2} \;,$$

$$F_{iii\,Q} = \frac{1}{(2a)^2}\{-[1+(3L/4a)^2]^{-3/2} + [1+(L/a)^2]^{-3/2}\} \;.$$

Expanding as before: $\quad F_{III\,P} \simeq -\frac{21}{128}\frac{L^2}{a^4}$, and: $\quad F_{III\,Q} = \frac{27}{128}\frac{L^2}{a^4}$.

Thus the sum of forces on the dipole pair is: $\quad F_{III} = \frac{3}{64}\frac{L^2}{a^4}$.

Nothing is changed from the first, directly opposite case! As long as our expansion approximation remains applicable, and we may say, for small α, β , the attraction exists, and in fact the same force exists from the source offset downward the same amount we considered an upward offset. Thus we must include a factor of two. By this logic the magnitude of the effect could be quite a bit larger than the single case I first analyzed. However, there is not a coherent stack of neatly consistent dipoles. According to the Sommerfield theory of electrons in metals, they have an average energy of a few electron volts, though according to a textbook [2] we should not expect this to be kinetic energy. Thus we will get a rough upper limit on average electron speeds if we calculate $1/2\, m_e v^2$ for say, one EV. This yields velocity of 6 E5 m/sec, and the time to travel one lattice cell length, taken as 0.3 nm, will be 0.5 E-15 sec.

The plasma frequency of silver has a photon equivalent wavelength of 137nm, and the time of transit for light will be that divided by *c*, or 4.6 E-16 sec. This is right about the same as electron transit time over one lattice distance, a surprising coincidence. Recalling that the electron's kinetic energy is not as large as the conduction band energy; and the text cited [3] mentions that energies in the Fermi-Dirac distribution do not represent kinetics, so we can say electron motion will be slower than the lightspeed signal of its position. There will thus be some opportunity for a positive reflection being induced directly across in the other plate. Also the dipoles will not be uniformly situated, so there is built into this system decoherence such that dipoles higher or lower will not contribute greatly to total force. We know that one dipole pair has a mutual force 1/60 of the Casimir, at a given separation. All it would take to raise this to equality would be four dipoles in each direction off-center with coherence with respect to the center dipole.

In a personal correspondence, H. Puthoff said to me he [4] thought the curve of attraction has no knee in it at the plasma frequency. If this is well-determined, it is a remarkable coincidence, since for higher-frequency vacuum fluctuations the metal is transparent. If one were to consider the sum of all forces on the assumption there was coherent geometry in the far, this sum of small discrete units may be treated as an integration. The surprising result is zero! We know that transverse dipoles attract their mirror opposite. Consider, however a dipole pointed toward the mirror with a colinear image. This will be repulsive. One can solve for the angle at which this force goes thru zero and it is $\arctan\sqrt{1/2}$ or 35.2 degrees. Based on an image separation of 2*a,* this is a vertical separation of $\sqrt{2}a$. Thus there are plenty of dipoles available at smaller angles though clearly the magnitude of the force decreases.

Mathematically one might expect that with a significant integration, the order of magnitude of the force might change toward inverse cube rather than inverse fourth power. In monopole statics, attraction of a charge to a long rod of charge goes as inverse first power; and indeed to a plate of charge, as the logarithm. Thus I predict that if the dipole effects are sufficient to keep the Casimir force curve smooth, then the curve should rise a bit slower, perhaps some fractional power between inverse third and fourth. If the effects are not such a match, there should be discernible a knee in the curve above the plasma frequency.

[1] Richtmeyer, Kennard, Introduction to Modern Physics, 3rd edition, 1942, McGraw-Hill.

[2] Albers, Casimir and Atomic Dipole Forces <paper under review at JETP.>

[3] Richtmeyer... (as above), p. 120

[4] email correspondence with H. Puthoff in September 2011.