FRIEDMAN AND DESITTER METRICS

N.Albers 7/25/8

With the assumption of a flat-space metric, $k = 0$, the Robertson-Walker field equations have the form:

$$\frac{4\pi G}{c^2}(\rho + 3\frac{P}{c^2}) = \Lambda - \frac{3R''}{c^2 R} \ ,$$

and:

$$\frac{4\pi G}{c^2}(\rho + \frac{P}{c^2}) = \frac{(R')^2 - RR''}{c^2 R^2} \ .$$

The Friedman solution assumes negligible pressure and no cosmologic constant, so the equations yield: $(R')^2 = \frac{D}{R}c^2$, where D is the geometric mass of the universe. If there is no pressure then we may say this is constant, or:

$$R^3 \rho = \text{const} \quad \text{and} \quad D = \frac{4}{3}\pi R^3 \rho (\frac{2G}{c^2}) \ .$$

The solution is: $R(t) = At^{2/3}$, $A = (\frac{3}{2}c)^{2/3} D^{1/3}$. The Hubble constant is: $H = \frac{R'}{R} = \frac{2}{3}t^{-1}$ and displays an asymptotic approach to zero.

On the other hand DeSitter solved the field equations assuming the state: $H = \text{const}$, so that: $H' = \frac{R''}{R} - (\frac{R'}{R})^2 = 0$. Including the cosmologic term and possible pressure:

$$\frac{4\pi G}{c^2}(\rho + \frac{3P}{c^2}) = \Lambda - \frac{3H_0{}^2}{c^2} \quad \text{and:}$$

$$\frac{4\pi G}{c^2}(\rho + \frac{P}{c^2}) = 0 \ .$$

The metric is simply expressed as: $R = R_0 e^{(Ht)}$.

Originally people thought that both mass-energy and pressure had to be positive or zero, but cosmologic expansion of a non-empty space cannot be accommodated this way. I propose the vacuum presents a field of negative energy but positive pressure. This is in harmony with a late-stage expansion where the Hubble constant of the expanding mass-energy of the Friedman solution has become small. This energy I call *adiabatic,* or ρ_A , as opposed to the residual vacuum energy posited as a negative constant. Thus: $\rho = \rho_A - \epsilon$, and we see pressure as:

$$\frac{P}{c^2} = \epsilon$$, where *epsilon* is a positive quantity.

We must be careful now to distinguish between the vacuum contribution and the adiabatic part. The conservation of total mass still holds true with respect to the latter. Indeed we must assume ρ_A gets asymptotically small, for then the regime of the DeSitter metric is appropriate. The first equation becomes now:

$$\frac{4\pi G}{c^2}(2\epsilon) = \Lambda - \frac{3R''}{c^2 R}$$. We are free to substitute for

the second derivative: $$\frac{8\pi G}{c^2}\epsilon = \Lambda - \frac{3}{c^2}\left(\frac{R'}{R}\right)^2$$. The last term contains the

square of H_0 , so this is the equation constraining the relationship of the three constants, vacuum pressure, cosmologic constant, and late-expansion Hubble constant.