KERR-SCHILD EM POTENTIAL

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The stated solution of the EFE's includes an EM 4-vector potential.

Looking at the first, scalar component, $A_0 = \dfrac{r^3}{r^4 + a^2 z^2}$.

We may move around “r's”: $A_0 = \dfrac{r}{r^2 + a^2 z^2 / r^2}$.

These are “Kerr coordinates” where r is the REAL part of complexified radius. Thus we recognize *z/r* as the redefined $\cos\theta$ for Kerr coords.

If one proceeds to do vector analysis, a decision must be made in which local basis vector system one wishes to make expressions. The Kerr-Schild system uses a Cartesian <x,y,z> basis to write the 'magick little vector field' which indeed characterizes A_i . Starting with the above electric potential, fields even in the tensor math are simply differentiated. We would add a time derivative, as the general expression is curl-like. To produce the entire

Minkowski field tensor, $F_{ab} = \dfrac{\partial \Phi_a}{\partial x^b} - \dfrac{\partial \Phi_b}{\partial x^a}$.

We choose time independence here, so electric field is a simple gradient calc.

We say: $E_a = -\dfrac{\partial A_0}{\partial x^a}$.

When one asks of the DIVERGENCE of this field, the rules involve the metric DETERMINANT. Here the mathematician must be aware and cautious. If we work in <x,y,z>, the DETERMINANT is constant everywhere. If we work in Kerr coords, $\sqrt{|g|} = (r^2 + a^2 \cos^2\theta) \sin\theta \equiv D$

and we can see the similarity with isotropic spheric coords. Call this quantity *D.* In a general metric space, we write the DIVERGENCE OPERATOR:

$$\mathrm{DIV}\,E = \nabla\cdot E = \frac{1}{D}\frac{\partial(D E_a)}{\partial x^a} \quad .$$

Now the business is calculating the fields and then their DIVERGENCE. I find the K-S results to be incorrect: their implied fields are:

$$E_r = -\frac{\partial A_0}{\partial r} \quad \text{and:} \quad E_\theta = -\frac{\partial A_0}{r\,\partial\theta} \quad .$$

I solve these as: $$E_r = \frac{r^2 - \cos^2\theta}{(r^2+\cos^2\theta)^2} \quad \text{and} \quad E_\theta = \frac{2\sin\theta\cos\theta}{(r^2+\cos^2\theta)^2} \quad .$$

Here I slip into unitizing a to save notation. Realize this, doing mental dimensional tests !!!

Here is where trouble brews, since these two should be the only fields, and their DIVERGENCE sum is not zero!!! Observe:

$$(D)\nabla\cdot E = \frac{\partial(D E_r)}{\partial r} + \frac{\partial(D E_\theta)}{r\,\partial\theta} \quad .$$

Having D in the numerator cancels one order of the mixed radial term below,

and we get: $$(D)\nabla\cdot E = \frac{\partial}{\partial r}\left(\frac{r^2 - \cos^2\theta}{r^2+\cos^2\theta}\right) + \frac{\partial}{r\,\partial\theta}\left(\frac{2\cos\theta\sin^2\theta}{r^2+\cos^2\theta}\right) \quad .$$

FAST FORWARD TO THE ALBERS FIELDS. If these rules of operation for the gradient and DIVERGENCES ops are correct, then apply the DIV operator to my proposed fields. Let us now simplify the radial part of the determinant, by defining: $R \equiv D/\sin\theta = r^2 + \cos^2\theta$. My fields read as:

$$E_r = R^{-1} + bR^{-1}r^{-1}P(\theta) \quad , \quad E_\theta = br^{-2}R^{-1}\sin\theta\cos\theta \quad .$$

I pieced these together to yield a zero sum in DIVERGENCE.