<u>FIELD MOMENTUM and INERTIAL MASS</u> 6/14/15

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Plotting lines of constant radius, or polar angle, in Kerr coordinates, yields the ellipses of *r*, and "scarab" lines perpendicular, for constant θ. We have expressions for electric field components in both directions in the near-field. The z-coordinate is in common with Cartesian maps, so we seek an expression for lines of constant θ to understand the local rotation of the Kerr basis with respect to our Cartesian 'picture'. With this in hand, we may integrate z-components of the momentum field.

In Richard Feynman's second volume of Lectures on Physics, he dedicates all of chapter 28 to discussing inertial mass, and field momentum. In flatspace, the calcs are of course simple. Two factors of sinθ arise with the same integral expression we constructed for field energy density, so the result is 2/3 of the former energy total integral. It should be ½, so this method betrays a problem in our simple former theory. With Kerr's metric solution applied in particle near-fields, we might expect different results, as angular symmetry is strongly altered.

I do not see the DKS fields to be useful; the complexified inverse radial form as potential function, does not yield good field energy totals. Thus I offer analysis with my proposed near-fields. Results are highly provocative, and slightly less than the integrated E-squared energy. In the second part of the study I present results with DKS fields.

We sense problems in accounting for field components in the 'equatorial' case. If we are looking down the SPIN-axis, integration over φ .is trivially 2π. To correctly write an *x*-component of field momentum, consider variation around the circle in φ. I have not completed a rigorous rotation argument, but it seems I have written what works. Consider first the 'axial' case, of momentum for an electron moving in *z*. In the Kerr geometry, we deal with both radial field components, and also in θ. Expressions are in Kerr's coordinates, as are the integrations we work with, $E_r \text{ and } E_\theta$, so I derived an expression for local coordinate system rotation. Consider coordinate lines of constant θ, at small radius. The locus of r=0, in a CARTESIAN MAP, is the disk, *z*=0, for radii in the *x-y* plane ≤ *a,* the characteristic angular momentum radius of Kerr. Except for right near the edge of this disk, these radial 'spokes' exit upwards or down, vertically. Then out a few radii, they have rotated to the usual far-field. Let us call the angle of the Kerr reference frame 'β', so in the far-field β=θ. Using $c \equiv \cos\theta$ one can show:

$$\sin^2\beta = \sin^2\theta \frac{r^2}{r^2+c^2} \qquad \text{and} \qquad \cos^2\beta = \cos^2\theta \frac{r^2+1}{r^2+c^2} \ .$$

Since the integral always has sinθ dθ = -d[cosθ], this is very useful and shows the symmetric behavior of the denominator polynomial in <*r,c*>, *except* for the important distinction of r_e not going to zero. The magnetic field generated as the particle moves along at a low velocity (non-relativistically) describes rings about the axis of motion. We are not here working with the intrinsic particle B-field. Since momentum density in a field is *E x B,* we analyze for the electric components orthogonal to *v,* first in the polar case:

$$P_z = v\left[\sin^2\beta\, E_r^2 + \cos^2\beta\, E_\theta^2\right] \ .$$

Since energy totals of the DKS field-squared show only the first term to be strong, look at: $P_Z = v\, r^2 (r^2+c^2)^{-1} E_r^2$. When an electric field has been realized as gradient of a potential, it is a covariant vector. Divergence is defined only on a contravariant field. When we square the field, we actually mean: $\tau = E_j E^j$, or starting with a field squared, $\tau = |E_k|^2 g^{kk}$ (no summation). The Kerr metric is diagonal except for the negligible terms in Schwarzschild *m.*

Here, $g_{11}=\frac{r^2+c^2}{r^2+1}$ and $g_{22}=r^2+c^2$ .We want to know now the inertial mass calculated in the orthogonal sense, for motion in the z=0 plane, say in *x.*

In the former classical calculations, we see the square of the sine, which yields integrals of 2/3. This simple radial field will calc the same, regardless of angle, of course, simply because velocity is the only preferred angle in the model. Cosine-squared yields the same as sine-squared when integrating by d(θ) but here we have d(cos) and 1/3. Thus we look at the Kerr near-fields which have information, dependence on polar angle θ. In this sense the field energy is an average of the two extremes in inertial mass calc. This is not realistic as a claim to physics, however, since solid angles give more angular space around the equatorial circle. A true spatial averaging would sum twice the second value, with the first, and give a value low by 1/3 of the difference.

Now we must admit the need to distinguish the equatorial case, integrating by φ. In the polar case this gives trivially, 2π. Not so in the sideways case, and analysis shows the following expressions to be useful:

$$[\sin^2\phi+\cos^2\beta\cos^2\phi]|E_r|^2 \quad + \quad [\sin^2\phi+\sin^2\beta\cos^2\phi]|E_\theta|^2 \;=\; \Pi_x \;\; .$$

This is an unusual expression, but it shows the different vector senses if, say, we plot going up to the pole, in the Φ=0 plane, or the Φ=π/2 plane. Since there is no other complication in this variable, each term integrates to ½ of its former value of 2π.

I offer some basic hints as to analysis on the polynomials in r^2+c^2 in the denominator. Whenever the numerator has either r^2 or c^2 , this clears one order from the denominator along with the numerator term. Always a factor of ½ is introduced, from “ $c\,\mathrm{d}c$ “. Only when the numerator is cleared is another factor of $\frac{1}{2r^2}$ drawn out of the polynomial under integration by *c.*

It may be vexing to Alexander Burinskii, but the final integration by c of one order of $1/(r^2+c^2)$ yields another order of 1/*r.* This integrates to $1/r \arctan 1/r$ and for small r this shelves at π/(2*r).* The proposed Albers fields do not suffer from this pathology. Let us examine them for energy totals.

$$E^r = \frac{1}{r^2+c^2}[1+\frac{\alpha}{r}P(\theta)] \quad \text{, and} \qquad E^\theta = \frac{\alpha}{r^2(r^2+c^2)}\sin\theta\cos\theta$$

where $P(\theta)$ is the second-order Legendre polynomial, equal to $3\cos^2\theta-1$. Squaring the field, the integrand along with the $d^3V = (r^2+c^2)\sin\theta\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi = -\ (r^2+c^2)\,\mathrm{d}r\,\mathrm{d}c\,\mathrm{d}\phi$ and the metric term appropriate to each, is: $\tau^r = [1+\frac{\alpha}{r}P(\theta)]^2$ and $\tau^\theta = \frac{\alpha^2}{r^4}\sin^2\theta\cos^2\theta$.

A knowledgeable reader can see the wisdom of my field constructions, here. In fact the radial component contributes very little near-field energy. Look at the second term: $\int_{r_e}^{\infty}\mathrm{d}r\int_0^1 \mathrm{d}c\,\alpha^2 r^{-4}s^2c^2 dc.$. Substitute for the sine, *s.*

$$\int_{r_e}^{\infty}\mathrm{d}r\int_0^1 \mathrm{d}c\,\alpha^2 r^{-4}(c^2-c^4) \ .$$

The result of having the terms in c^n is a simple difference of fractions, so this is easily analyzed to: $1/3\,r^{-3}(1/3-1/5) = \frac{2\alpha^2}{45}r_e^{-3}$.

Nominally the inner radius is taken as the classical energy radius. Thus the number I have written before, 22.5.

We want to know now the inertial mass calculated for motion on the axis of spin: $$P_z = |E_r^2|\sin^2\beta + |E_\theta|^2\cos^2\beta \quad .$$

In the former classical calculations, we see the square of the sine, which yields integrals of 2/3. This simple radial field will calc the same, regardless of angle of the velocity vector, of course, simply because velocity is the only preferred angle in the model. Cosine-squared yields the same answer when integrating by d(θ) but here we have d(cos) and 1/3. We must further analyze geometry in the z=o plane; now there is need to distinguish the equatorial case, integrating by Φ. In the polar case this gives trivially, 2π. Not so in the sideways case, and analysis shows the following expressions to be useful:

$$[\sin^2\phi + \cos^2\beta\cos^2\phi]|E_r|^2 \quad \mathbf{+} \quad [\sin^2\phi + \sin^2\beta\cos^2\phi]|E_\theta|^2 \;=\; \Pi_x \quad .$$

This is an unusual expression, but it shows the different vector senses if, say, we plot going up to the pole, in the Φ= 0 plane, or the Φ=π/2 plane. I write β rather than θ to allow for the generalized Kerr geometry. One can see the classical result by realizing there is no θ-field. Both integrations around the circle yield π, or ½ the previous result of 2π. Thus we recover ½ [1+ 1/3] or 2/3 as per Feynman, and have isotropy in the simple flatspace case.

Let us consider now Kerr geometry, and start with polar momentum, which is to say motion of the electron along its spin axis:

$$P_z = v[\sin^2\beta\, E_r^2 + \cos^2\beta\, E_\theta^2] \quad .$$

With the Albers fields, sense that both terms may contribute, since multiplication by the cosine decreases totals, but the θ-component is stronger. Analyzing both,

$$P_z = \int \mathrm{d}r \int \mathrm{d}c \left[\frac{r^2}{r^2+c^2} \sin^2\theta [1+\frac{\alpha}{r}P]^2 \; + \; \frac{r^2+1}{r^2+c^2} \cos^2\theta \frac{\alpha^2}{r^4} s^2 c^2 \right] \; .$$

The first integral may well be approximated by: $\mathrm{I} = \int \mathrm{d}r \int \mathrm{d}c \frac{r^2}{r^2+c^2} \sin^2\theta$,

knowing only the near-field contributions matter. One can detail the calcs with the denominator term, but the *r*-squared in the numerator dominates, and the integral is “quite small”.

The second integrand is more subtle: $\frac{\alpha^2}{r^4(r^2+c^2)} s^2 c^4$.

We know to deal with the *s*- and *c*-terms in θ, but given the denominator,

$$\frac{\alpha^2}{r^4} \int \mathrm{d}c \, \frac{s^2 c^4}{r^2+c^2} \quad \text{becomes} \quad \frac{\alpha^2}{r^4} \int \mathrm{d}c \frac{c^4 - c^6}{r^2+c^2} \; .$$

On the first reduction in *c* integrating by parts, we see:

$$\int \mathrm{d}c \frac{c^4}{r^2+c^2} = \frac{1}{2} \log(r^2+1) - \frac{3}{2} \int c^2 \log(r^2+c^2) \; ,$$

This logarithmic argument moves slowly at small *r*, (unless c=0) so looking in the CRC integral tables, it is gratifying to see other terms drop away, leaving only a '2'. The strongest term in this integral is ½ , so we write:

$$\frac{3}{4} \alpha^2 \int \mathrm{d}r \, r^{-4} = \frac{\alpha^2}{4} r_e^{-3} \; .$$

Now let us figure the second term. Again reducing 2 orders of *c*,

$$\int \mathrm{d}c \, c^6 (r^2+c^2)^{-1} = \frac{5}{2} \int \mathrm{d}c \, c^4 \log(r^2+c^2)$$

With another integration by parts: $I = \frac{5}{2} \cdot \frac{3}{2} (1/2) \int \mathrm{d}c \, c^2 = 5/8$.

As before the integration over *r* yields a 1/3 , so we see a result of 5/24. Subtracting this from the first result, we are left with $\alpha^2/24 r_e^3$. This is only slightly less than the field energy result of 2/45 !!

Now let us construct the equatorial inertial result. We can see the logic of the form, on page 3 above. It can be shown the radial field component contributes very little, so look at the second part, with $|E^{\theta}|^2$. We see the first term in $\sin^2\phi$ is the perfect result, namely ½ of the field energy integral. Is the second term small? $\cos^2\phi\,\sin^2\theta\,|E^{\theta}|^2$. We know the integration around the circle is trivial, so:

$$\frac{\alpha^2}{r^4}\int \mathrm{d}c \frac{r^2}{r^2+c^2} s^4 c^2 = \frac{\alpha^2}{r^4}\int \mathrm{d}c \frac{r^2}{r^2+c^2}(1 - 2\mathrm{c}^2\, 1 + 1\, c^4)c^2 = \frac{\alpha^2}{r^4}\int \mathrm{d}c \frac{r^2}{r^2+c^2}(c^2 - 2\mathrm{c}^4 + c^6)$$

We may combine terms in r : $$\ldots = \frac{\alpha^2}{r^2}\int \mathrm{d}c (r^2+c^2)^{-1}(c^2 - 2\mathrm{c}^4 + c^6) \;.$$

Procedure is as before: $\int \mathrm{d}c\, c^2 (r^2+c^2)^{-1} = 1/2 \int \mathrm{d}c \log(r^2+c^2)$; etc.
However, we see now only $1/r^2$ and realize no other orders of r are forthcoming here, from integrations over *c.* Thus our job is finished, as we see this is a very small quantity. Have a beer !!!