ELECTRON NEAR-FIELD in the KERR METRIC

N. Albers 10/2/13

email: nvalbers@gmail.com

I dedicate this study to Ludwig Boltzmann.

Rather than relegating divergence of the electric field to a point of charge, posit a thickening near-field of charge density described by:

$$\rho = q_0 \frac{e^{-r}}{r} \quad .$$

This radial coordinate shall be scaled to match the real energy and charge, along with q_0 . We treat this as a vacuum manifestation and ascribe circulation at the speed of light, with a somewhat obvious angle factor:

$$j = \rho c \sin\theta \quad .$$

In my 2005 study I describe solution consistent with Maxwell's equations, where the job is to solve for magnetic vector potential. The electric component is easily integrated, and tho charge density peaks as inverse radius at the center, there are no infinities or residues when we integrate for observable quantities. This is the genius of the math expression, also used by Yukawa to represent the strong force. We get a near-field falling away in the far, to be scaled for whatever problem is being considered.

The magnetic vector potential consists only of A_ϕ , following the circular currents posited. It can be expressed as:

$$A_\phi = \frac{2}{3} r^{-2} - \frac{1}{3} e^{-r} (2\mathrm{r}^{-2} + 2\mathrm{r}^{-1} + 1) - \frac{1}{3} [(\int dr \, \frac{e^{-r}}{r}) \ - \gamma] \quad .$$

All expressions have implied constants in front of them, which we leave out for now, as described at the start.

I am inspired by Alexander Burinskii's electron model where the outer electric and gravitational field, i.e. the Kerr solution, yields entirely at the classical energy radius, to a false vacuum of string theoretic description. My inhomogeneous near-field may similary be 'plugged in' since it is indeed scaled at about the classical radius. This approach also answers to obviating the embarrassing orders of infinity, as the E-field is increasingly 'shaded down' going inward and integrations for charge and for field energies are indeed calmly behaved and finite. The model describes a steady “thickening” of the vacuum and electric permittivity rises asymptotically toward the center.

The Kerr metric solution is not simple and working with the determinant is at first daunting and difficult. My original study was analyzed in 4-vector potential field equations, but now we must work in tensor form. My source may be simply written: $s^a = \frac{e^{-r}}{r}[1,\ 0,\ 0,\ \sin\theta]$. Note the presumed speed 'c' cancels since we would write *j/c.* Maxwell's equations are succinctly expressed: $\frac{\partial(\sqrt{-g}\,F^{ab})}{\sqrt{-g}\,\partial x^b} = s^a$, and for the electric field:

$$\frac{\partial E_r}{\partial r} + \frac{\partial\sqrt{-g}}{\sqrt{-g}\ \partial r}\ E_r = \frac{e^{-r}}{r} \quad .$$

Now we state the Kerr metric:

$$ds^s = g_{00}(dx^0)^2 + g_{11}(dr)^2 + g_{22}(d\theta)^2 + g_{33}(d\phi)^2 + g_{03}dx^0 d\phi$$

with: $g_{00} = 1 - \frac{2\mathrm{mr}}{r^2+a^2\cos^2\theta}$, $g_{11} = -\frac{r^2+a^2\cos^2\theta}{r^2+a^2-2\mathrm{mr}}$, $g_{22} = -(r^2+a^2\cos^2\theta)$,

$$g_{33} = -\left[(r^2+a^2)\sin^2\theta + \frac{2\mathrm{mra}^2\sin^4\theta}{r^2+a^2\cos^2\theta}\right] \text{, and} \qquad g_{03} = -\frac{4\mathrm{mra}^2\sin^2\theta}{r^2+a^2\cos^2\theta} \quad .$$

We need to state the determinant, which is $g_{11}g_{22}(g_{00}g_{33} + g_{03}^2)$:

$$-g = (r^2+a^2\cos^2\theta)^2\sin^2\theta \quad .$$

One may express the derivative of the logarithm. Call 'D' the square root of *-g,* and note the factor of 2. We drop terms in '*m'* since it is so small.

$$\Gamma \equiv r\frac{\partial D}{D\partial r} = r/2\frac{\partial g}{g\partial r} = \frac{2r^2}{r^2+\cos^2\theta} \quad .$$

Note the r on the LHS, for simpler writing. In the far field we expect a result of simply 2, and this is the case.

When the vacuum is deemed 'empty', there is no source term, and we may divide out the 'E': $$\frac{dlnE}{dr}+\frac{dlnD}{dr}=0 \quad .$$

This gives simply, $E = D^{-1}$ within a constant multiplier; the θ-dependence drops away, as discussed below, on page 5.

The cosine term equals one, on the z-pole, so let us analyze behavior at different distances.

$$\Gamma_{pole} = \frac{2r^2}{1+r^2} \quad .$$

The far-field yields the value '2', and in the near approaches zero! We can see this holds on the entire disk, characterized by $r = 0$. At the edge of the ring also the cosine goes to zero, so we must analyze with care. As we approach the disk from the outside, the limits are clear, since identically $\theta = \pi/2$, and $r>0$. Thus the exponent is the far-field value. As we range up or down, tho, we again see dependence dropping to zero; there is a functional balancing act with small 'r' and small $\cos\theta$. Thus in a 'small' radius around the edge there is a transition region, near the disk.

We may then describe a lobe, a locus of $\Gamma = 1$. It includes the outer edge point, and is solved simply as $r = \cos\theta$, which intersects the pole. At the mid-angle, $\pi/4$, a value of 1 is reached at radius slightly smaller than $1/\sqrt{2}$.

The classical energy radius is $a/137$, so this phenomenology is strong before this smaller dimension is reached. Yet the existence of this spacetime field depends on our assumption of a source. Burinskii excises the inner region utterly, proclaiming a false vacuum of string theoretic characteristics. In late-breaking results, he informs me there is another component to the electric field ! Examine the vacuum divergence equation:

$$\nabla \cdot E = 0 = D^{-1}\frac{\partial(DE^b)}{\partial x^b} \quad .$$

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If we look for a *θ*-component, then we have a sum of two terms:

$$\frac{\partial(DE_r)}{\partial r} + \frac{\partial(DE_\theta)}{r\,\partial\theta} = 0 \quad .$$

This seems to be a difficult situation, since there are two field variables, but only one constraining equation. If we retain the validity of the radial part, however, acknowledging the integration *by 'r',* which yields a constant on the RHS, the constant may also be a function of θ. We want no further angular dependence in the far-field, and yet the far-field determinant contains $\sin^2\theta$. We are free to divide this out.

We must have the second term also zero, and the same argument applies except we are now integrating by θ, so the RHS 'constant' may include any arbitrary function of radius. Other than that it is also proportional to the inverse of D! Our physics dictates this falling off in the far-field. We know also that near-field falloff in the radial part shows the need for stronger fields here, and in fact two orders of inverse '*r*' are called for. Posit an angular component of: $E_\theta = r^{-2}D^{-1}$, although here the denominator contains sinθ, and this, when squared for energy density, is not finite under integration. We must find a further program of logic. Our radial far-field is successful, but perhaps a lower-order term could be added to it so the sum of divergence terms, along with a changed θ-dependence for that component, yields zero.

We took our initial radial form to satisfy: $D E_r = f(\theta)$,

but find the inverse sine function not tolerable so let us assume it gone !! Rather a more reasonable function would have this dependence in the numerator, so the pole itself does not offer pathologic behavior. Since the determinant also has this term: $E_\theta \equiv (r^2 + \cos^2\theta)^{-1}\sin\theta$ and $DE_\theta = \sin^2\theta$, so the derivative of this by θ gives:

$$\frac{\partial(DE_\theta)}{\partial\theta} = 2\sin\theta\cos\theta \quad .$$

Looking at the sum of terms on the previous page, we may divide by the sine:

$$\frac{\partial(DE_r)}{\sin\theta\,\partial r} + \frac{2}{r}\cos\theta = 0 \quad .$$

We ask what may be added to the first term? We know the second term would satisfy the differential constraints with any arbitrary 'coefficient' in *r.* Let us create an added radial term, knowing that our first solution is valid and yields zero to the differential operator: $\frac{\partial(DE_q)}{\partial r}$, so a further term must add to the second term to yield zero. We know the second term may be freely adjusted in the radial coefficient, as the first term may be adjusted in the θ-dependence. We choose: $E_q = 2r^{-1}D^{-1}\sin\theta\cos\theta$ and also, $E_\theta = D^{-1}r^{-1}\sin^2\theta$, since we have been looking ahead to what is needed! The first \expression is one order in *r* stronger in the near-field, and the second is two orders more.

Examining the sense of these fields, we see the θ-component would be better rendered as a cosine, for symmetry. Likewise the cosine is not welcome in our radial part. Let us use: $DE_\theta = r^{-1}\sin\theta\cos\theta$.

This gives terms symmetric about the *z*=0 plane: $r\frac{\partial(DE_\theta)}{\partial\theta} \approx \cos^2\theta - \sin^2\theta$, so we choose: $E_q = r^{-1} D^{-1} \sin\theta(\cos^2\theta - \sin^2\theta)$. There is yet a problem balancing the $\sin\theta$ of the radial addition. Let us use instead for the component in θ : $$r\frac{\partial(DE_\theta)}{\partial\theta} \approx \sin\theta(2\cos^2\theta - \sin^2\theta) \ .$$

It is now clear what the radial addition should be. To summarize the fields we have constructed:

$$E_r = (r^2+\cos^2\theta)^{-1} + \alpha r^{-1}(r^2+\cos^2\theta)^{-1}(2\cos^2\theta - \sin^2\theta) \ ,$$ and

$$E_\theta = \alpha r^{-1}(r^2+\cos^2\theta)^{-1}\sin\theta\cos\theta \ .$$

At the inner limit of E-2 or the classical energy radius, we see the added radial part overwhelms the oppositely-oriented original term. We may however scale together the added components, E_q and E_θ , by the same constant. A first effort will then be to make the radial terms equal and opposite at the inner limit, as shown by including α as a coefficient. The fine structure constant is indeed the ratio of the classical energy radius, and the angular momentum radius, *'a'.*

We square the orthogonal components and add the results. I show the analysis of the last radial term, which can be better traced if we alter the form of the added radial part: $2\cos^2\theta - \sin^2\theta = 3\cos^2\theta - 1$. The integrations must include a coefficient on the differential volume element, of D, and are:

$$T = \int_{r_e}^{\infty} dr \int_0^{\pi} d\theta\ (r^2+\cos^2\theta)\sin\theta\ \ \alpha^2 r^{-2}(3\cos^2\theta-1)^2(r^2+\cos^2\theta)^{-2} \ .$$

Observing that: $d(\cos\theta) = -\sin\theta(d\theta)$, examine the last term of the three in squaring the Legender term: $T = \int\int dr\, dc\,(r^2+c^2)r^{-2}(r^2+c^2)^{-2}$

Here I confess to appreciating the CRC integral tables for the *c*-integration:

$$T(r) = \alpha^2 \int_{r_e}^{\infty} dr\, r^{-2}[\frac{c}{r}\tan^{-1}\frac{c}{r}] \ ,$$ evaluated in *c*.

We get contribution at *c*=1, so: $T = \alpha^2 \int dr\, r^{-2}[r^{-1}\tan^{-1} r^{-1}]$

The inverse tangent at small '*r*' is *π*/2, so the significant integration in '*r*' is:

$$T = \frac{\pi}{2}\alpha^2 \int_{r_e}^{\infty} dr\, r^{-3} = \frac{\pi}{4}\,\frac{\alpha^2}{r_e^2} \ .$$

Any order of $\frac{\alpha}{r_e}$ equals 1.

Completing the other terms, we see we have done *too well.* We have cancelled out the radial field where it was strong, so to this order of terms, the radial field shows only small energy totals!!! The original radial term integrates to $\pi \ln r_e$, but the cross-term of the two parts equals -π. Back to our drawing boards. The first energy term is dependent on inner radius, without α . This changes only slowly so it makes not a large difference in the final sum.

The integration of E_θ energy yields a value if π/4, and the total energy is: $T = \pi(\ln r_e^{-1} - 1/4)$.

With $r_e = 1/100$, the logarithm equals 4.7. There are two avenues available for this theoretic thrust. Any constant multiplier where we have put α, cannot be any smaller. We have need to make it larger, and this will succeed, in fact, embarrassingly! The number needed is very near 2π. All four of the integrals relevant to the present sum are simple fractions of π, and I can write a quadratic equation in 'α' and it shows the above number works. It would be perhaps about 2π, but in a separate short study I have figured the magnetic energy of a superconducting current loop at r_e to be 8π. The actual fine structure constant *in this entire exercise* is figured at 1/100 rather than 1/137, just for easy numbers. I believe this is not intrinsic to the logic of the maths. The only assumptions have involved statements like, $r_e = \alpha a \ll a$ where *a* is angular momentum radius, roughly E-13 meters. We are accustomed to defining the classical energy radius according to total particle energy and electric field rules. It should read as: $4\pi / r_e$, in a 'unitized' presentation such as here; electric charge is seen as: $q = 1 = e/4\pi\epsilon_0$.

I simply do not include here, the symmetric 2π integration in φ. Thus my result should say TOTAL ENERGY=200. We could subtract the magnetic energy of 8π to look for an electric total of ≈175. However it seems the classical radius is simply defined in terms of electric field, even if this is not a real, specific quantity. The present arithmetic attack is not a change in the FSC; we are free to multiply our 'inhomogeneous' added field terms by the *same arbitrary* constant.

The other avenue would be to choose a higher order exponent for the inverse '*r'.* Since energy density is the square of the field, this rapidly increases totals, and it looks like a fractional power would be needed. The path I now take is the simpler. It will be noted the radial field just outside the ring goes *through zero* and actually is reversed. This is manifest in a narrow sheath, near the outer part of the disk. At the disk, Cartesian radial changes map into the *θ*-variable, so in as far as the Legender term is negative, or: $3\cos^2\theta = 1$, this is so. Later I shall present the locus where radial field goes through zero.

Let us condense our notation, so that: $P \equiv 3\cos^2\theta - 1$, and radially: $R^2 \equiv r^2 + \cos^2\theta$, Total energy has four parts, and the fields are now: $E_r = R^{-2}[1 + P\alpha/r]$, and: $E_\theta = R^{-2}(\alpha/r^2)\sin\theta\cos\theta$. Squaring both components, we see the radial parts produce three terms, and the angular part, one. The total energy integrates to:

$$T = \pi\left[-\log r_e - \frac{\alpha}{r_e} + \frac{3}{4}\left(\frac{\alpha}{r_e}\right)^2\right] \ .$$

Let us increase the arbitrary constant along with α by a factor *'k'*:

$$T = \pi[-\log r_e - k + 3/4\, k^2] \ ,$$

We acknowledge the ratio with the FSC to be 1. We can see that if $k = 2\pi$, our total is only about 60, so we need a larger number still. This is simply a numbers game, and if we choose *k*=9, we get:

$$T = \pi[4.7 - 9 + 61] \approx 180 \ .$$

This is a comfortably close figure, and if we evaluate k=10, this yields more than 200. My feeling is that the classical energy radius is not a physically measured quantity, and we know there is some magnetic energy as well as electric, although it seems to be much smaller. A number slightly larger than 9 works, and 3π is within a half-percent.

In my original inhomogeneous model, magnetic energy was a bit larger than electric field energy. My presumed charge cloud distribution produces this relationship, but I posited increasing charge and current going inward. If my first attempt to calculate magnetic energy of the superconducting loop in the present model is correct, it is 8π, or about 25, except there is a surfeit of terms in r_e in the numerator, and this makes it clear the magnetic total is small.

At this point, it is salutary to examine the Kerr coordinates near the ring. Recall that the entire disk is characterized by r=0. The ring lies at a Cartesian radius of '*a*'. Just outside and in the z=0 plane, $\cos\theta = \sigma = 0.$, so:

$$r^2 = r_C^2 - a^2 = r_C^2 - 1. \ .$$

If r = 9/100, then: $r_C^2 = 1 + 0.0081$, and: $r_C = 1.004$. We see the Cartesian radius increases much more slowly here; a Kerr radius of 9 times r_e is only 0.4 r_e in Cartesian space.

It is also useful to get a sense of the magnetic energy. Without any detailing of angular distributions, we may consider a “very thin ring” of current at the inner radius and represent it by a Dirac delta function. Given a charge '*e*' circulating around the ring, we may represent the current: $j = ec/2\pi r_e$. This will be the RHS source term for the magnetic vector potential A_ϕ , which is solved by the method of variation of parameters. We know the homogeneous roots: < r^{-2}, r > and their Wronskian is $-3r^{-2}$. Thus we may construct the inhomogeneous form:

$$A_\phi = (-1/3)\, r^{-2} \int dr\, r\, r^2 \delta(r - r_e) = -\frac{r_e^3}{3r^2} \quad .$$

This is the only relevant root, since the other one represents a constant magnetic field.

Magnetic source energy density may be stated: $A_\phi \cdot j$,

and its total may be integrated: $\Xi = \int dr\, r^2 A_\phi \cdot j$.

Again the delta function selects the inner radius, and we see that:

$$\Xi = -\frac{ec}{6\pi} r_e^2 \quad .$$

It is clear this is a very small quantity compared to unity, since the inner radius equals the AM radius divided by 100 (or by the inverse FSC).