ELECTRON FIELD SOLUTION WITH CIRCULAR CURRENTS

by Norman Albers

PART I: Field Solutions of the Electron

Electromagnetic theory has been incapable of modeling the electron as a field because it represents point charges in a vacuum. Offered here is a reasonable and mathematically minimal construction of inhomogeneous charge and current terms, added to the usual far-field. Thus the severity of the singularity is limited and now fully integrable. The existence of a static, circular mode of solution is proposed.

It is thought by most that quantum mechanics comprises all that may be said about the fundamental quanta. A self-consistent construction assumes an inhomogeneous spherical charge, and circular current field. Working in spherical coordinates, one finds that only A_φ generates reasonable behaviors at the origin. For the same reason there must be no time dependence in scalar potential, U. If field strengths go as r^{1} at the origin then observables have finite integrals, as they (energy, angular momentum, etc.) go as r^2. This motivated the mathematical winnowing process. The physics is that of a static charge-current assembly with a factor of ½ for correct accounting of energy interaction terms. "Static" means "unchanging in time" and so includes momentum and current circulating steadily around the z-axis.

The complete current equation is

$$\Box A + \nabla(\nabla\cdot A) + c^{-2}\nabla(\delta U/\delta t) = j/\epsilon_o c^2$$

If U is static,

$$\nabla\times\left(\nabla\times\mathbf{A}\right) + c^{-2}\ddot{A} = \mathbf{j} \qquad \left(\varepsilon_0 c^2 \equiv 1\right).$$

Assuming $\mathbf{A}$ is only $\hat{\varphi}A_\varphi$, and that $\dot{\mathbf{A}} = 0$,

$$\nabla_r^2 A_\varphi + \nabla_\theta^2 A_\varphi - r^{-2}\sin^{-2}\theta A_\varphi = -j.$$

Choose θ-dependence of $\sin\theta$, or $P_1^1(\theta)$. The θ-operator resolves:

$$\nabla_r^2 A_\varphi - 2r^{-2}A_\varphi = -j.$$

This is the relevant current equation. Posit now an inhomogeneous, static charge/potential field:

$$U = -U_0 r^{-1}\left(1 - e^{-r}\right).$$

The combination of terms makes manageable the singularity, and $r^{-1}e^{-r}$ gives a charge density of:

$$\rho = -\nabla^2 U = -U_0 r^{-1} e^{-r}.$$

Treat this as moving locally at the speed of light in $\hat{\varphi}$: $(U_0 \equiv 1)$

$$j_\varphi = \pm c\rho \sin\theta,$$

where the sign is chosen for up-down consideration. This is justified if the momentum of the current mode is taken as $\rho\mathbf{A}$. If we could multiply momentum by charge/mass, we should have current. Use $c^{-2}\mathbf{j}\cdot\mathbf{A}$ as the mass-energy:

$$\mathbf{j} = \rho\mathbf{A}\left(\frac{\rho c^2}{\mathbf{j}\cdot\mathbf{A}}\right) = \frac{\rho^2 c^2}{j}\hat{\mathbf{A}}.$$

Thus, $j = \pm\rho c$, in $\hat{\mathbf{A}}$.

We can see that a positron will also have positive energy: the sign of the current determines the sign of $\mathbf{A}$, and only ρ changes to positive. I offer no physical justification for choosing mass-energy so; this is the only analytically soluble case and is thus useful.

Take a positive current as the source term of the inhomogeneous current equation,

$$\nabla_r^2 A_\varphi - 2r^{-2}A_\varphi = -cr^{-1}e^{-r}\sin\theta.$$

Even though these modes are unchanging in time, a Poynting flow can be construed as:

$$\mathbf{E}_r \times \mathbf{B}_\theta = \mathbf{P}_\varphi.$$

We are generating a magnet with near and far fields, opposites. Thus $\mathbf{P}$ changes direction. We chose to identify one direction for $\mathbf{j}$, and this is consistent with A_φ being positive-definite:

$$A_\varphi \sin^{-1}\theta = \frac{2}{3}r^{-2} - \frac{1}{3}\left(2r^{-2} + 2r^{-1} + 1\right)e^{-r} - \frac{1}{3}r\left[\int r^{-1}e^{-r}dr - \gamma\right].$$

The homogeneous term, in r^{-2}, is put in to cancel the singularity. At the origin, field strengths go as r^{-1}; densities of conserved quantities go as r^{-2}, and integrate without singularity when multiplied by r^2 in the volume element.

This model yields a fine-structure constant of roughly unity, to be explained elsewhere. All quantities have been integrated to the origin, *with no cutoff*! Since energy density continues to climb as r^{-2} inside the classical radius, and was already roughly 10^9 gm/cc, neutron star density of 10^{16} gm/cc is surpassed within four magnitudes of reduction in r. Beyond here, and without a massive core there is no reason to be stopped at the phase changes distinguishing phases of stellar masses, one must clearly have a relativistic model [1]. Density can be seen to rise to immense values as r approaches the Planck length, though there is no central spike of total energy, charge, or angular momentum.

The mathematics is identical to the superconducting solution in a material (London and London [2]; Meissner [3]), except that they posit zero charge accumulation. Solving A_φ under this assumption,

$$\nabla_r^2 A_\varphi - 2r^{-2}A_\varphi = \lambda^2 A_\varphi.$$

This is a homogeneous equation, solved by: ($\lambda \equiv 1$)

$$A_\varphi = \left(r^{-2} + r^{-1}\right)e^{-r}.$$

The same terms are seen shifted around after we let this current be seen as a charge field, and solve for the inhomogeneous part of the scalar potential U:

$$U = r^{-1}e^{-r} + \int r^{-1}e^{-r}dr - \gamma.$$

The electron is thus clearly seen as a "superconducting" spherical cloud by virtue of being the sum of homogeneous and inhomogeneous fields. From his vantage point 250 years ago, Leonhard Euler receives his due.

PART II: Dielectric Interpretation of Electrons

Given a charge distribution of:

$$\rho = -r^{-1} e^{-r} \quad ,$$

we are free to interpret its source. If we think of it as a monopolar density we identify: $\rho \equiv \nabla \cdot \boldsymbol{E}$,

then we could integrate for ***E***, and get:

$$E = (r^{-2} + r^{-1}) e^{-r} \quad .$$

We may imagine now a polarization field ***P*** with divergence such that it accounts for the charge density: $-\nabla \cdot \boldsymbol{P} = \rho$.

This is saying ***P*** is equal and opposite to the inhomogeneous part of the electric field. To complete *E,* however, add a homogeneous term to balance the singularity in r^{-2} : $E = -r^{-2} + (r^{-2} + r^{-1}) e^{-r}$.

Now we can look at P/E to get to permittivity:

$$\epsilon_h - 1 = P/E = N/(1-N) \text{ , where } N \equiv (1+r) e^{-r} \quad .$$

This yields: $\epsilon_h = 1/(1-N)$,

and a physically interesting model. Electric field can be expressed:

$$E = -r^{-2}(1-N) \quad .$$

Taking the limit at the origin, the behavior of permittivity is:

$$\lim_{r \to 0} \epsilon_h = (1/2) r^{-2} \quad .$$

The speed of light is the inverse square root , or:

$$\lim_{r \to 0} c / \sqrt{(\epsilon_h)} = r\sqrt{2} \quad ,$$

and tends to zero at the center. This preserves a coherent circulation of energy, since despite the assumption of no $\delta A_\varphi / \delta \varphi$, energy does flow in $\hat{\varphi}$ and imagined wavefronts pivot. This is another phase state of light, just as ice is another form of water with a different physics of its constituents. There is no further need to explain mass. If we see how energy is "convinced" to spin in a small locale, there is no further question if it manifests the electric and magnetic fields of the electron/positron. The physics being illuminated here is of the dipole contribution from the vacuum. Whether the picture of quantum mechanics of virtual "particles" is more accurate than one of space as a more infinitesimal sea of available inhomogeneous fluctuations, there must be the manifestation of charge and current. In a concurrent paper on Photon Localization, I show how such physics allows the existence of localized wave packets. That analysis can be applied directly to vacuum fluctuations to reveal non-quantized charge densities, such as are needed here. Regardless, we can speculate on some fascinating possibilities. If we picture a dipole pair, and it points outward in a negative electron field, the particles will be drawn back together to annihilation. Those pointing inward, parallel to the total electric field (we defined it this way, since the inhomogeneous part is smaller than the homogeneous), will be tugged apart somewhat. We can see a natural selection in harmony with the stability of this state of energy. Furthermore, the positive end will be closer to the center and feel a slightly stronger electric field, so it is more strongly attracted than the negative end is repelled. Thus, the dipole as a unit experiences an attraction toward the center. This is a remarkable state of affairs for a system which, viewed as a classical "assembly of charge", should want to fly apart. There will be a diffusion of dipoles inward; they cross vertical lines of magnetic field, and this turns the two particles in opposite directions sideways, or into $\hat{\varphi}$, contributing to the circular currents which must exist. Beyond that, we can say that there is a negative dipole pressure, as they are attracted inward. This is notably important especially for the relativistic solution needed at very high energy densities near the singularity. Reminiscent of Higgs theory which depends on negative pressure, this is presumably a manifestation different from Higgs bosons.

[1] Adler, Bazin, Schiffer, Intro. to Gen. Rel., McGraw-Hill (1965), p. 467.

[2] H. London, F. London, Proc. Roy. Soc. (London), A149, 71 (1935).

[3] W. Meissner, R. Ochsenfeld, Naturwiss. 21, 787 (1933).

PART III: Effective charge

In a field of varying permittivity we expect to experience an electric charge by the relationship: $E = q/(4\pi\epsilon r^2)$. Let us look at charge as the effective measurement experienced in this electron field, and write:

$$q_{eff} = 4\pi\epsilon r^2 E \quad .$$

In the limit at the origin the electric field goes to *-1/2,* as can be seen by expanding the exponentials in the expression for *E*. Permittivity goes as the inverse square of radius so these terms cancel. This says that the observer always feels the same charge is yet present at the center even though part of the inhomogeneity lies outside the region of interaction.

PART IV: Magnetic Moment Interaction Integrals

The electron is modeled as an inhomogeneous charge distribution: $\rho = r^{-1}e^{-r}$, and current is taken as: $j = \pm\hat{\phi}\rho sin(\theta)$. Let j be positive for the spin-down case, with magnetic moment in $+\hat{z}$. We may integrate the energy of a magnetic interaction with a relatively weak and locally uniform 'lab' field by integrating the z-component of a spin-aligned state:

$\mu = \int_0^\infty B_z d^3V$. This will yield the contribution from the fields, and later we express source terms. Starting with the vector potential of:

$$A_\phi = 1/3 sin\theta[2r^{-2} - (2r^{-2} + 2r^{-1} + 1)e^{-r} - r[\int r^{-1}e^{-r} - \gamma]] \quad ,$$

we take the curl to produce the B-field in spherical coordinates:

$$B = +\hat{r}(rsin\theta)^{-1}\partial_\theta(sin\theta A_\phi) - \hat{\theta}(r)^{-1}\partial_r(rA_\phi) \quad .$$

Projecting the z-component and completing the differentiation in *θ,*

$$B_z = -B_\theta sin\theta + B_r cos\theta = sin\theta\, r^{-1}\partial_r(rA_\phi) + 2cos^2\theta\, sin^{-1}\theta\, r^{-1}A_\phi \quad .$$

Now let us write only the radial dependence: $A \equiv A_\phi(r)$, and collect terms: $B_z = \sin^2\theta r^{-1} \partial_r(rA) + 2\cos^2\theta A$.

We may now complete the angular integrations, with their implied $\sin\theta \partial\theta$, and write: $B_z = 4/3 r^{-1} [\partial_r(rA) + A]$.

For the final integration radially: $B_{zT} = \int_0^\infty dr(r^2) B_z$, but we integrate the first term by parts:

$$B_{zT} = 4/3 [\int_0^\infty dr(r\partial_r(rA) + rA)] = 4/3 [r^2 A|_0^\infty - \int_0^\infty (-rA + rA)] .$$

Thus we are left with only a kernel to be evaluated on both a vanishingly small interior radius and the far sphere (only the $\int_0^{2\pi} d\Phi$ is not yet expressed). Any finite magnetic dipole has a far field of: $\lim_{r\to\infty} A \simeq r^{-2}$, and I describe a near field whose contribution is of higher order and thus vanishes. These are the two constraints on any such electrodynamic theory, and reflect aspects of divergence analogous to the electric field. One should not interpret too literally, however, since we are distilling only the radial function of the azimuthal vector component here, and the total vector field has zero divergence since it consists only of A_ϕ with no variation in *Φ.* Thus we have a simple rule to evaluate the field contribution, and given the far limit of *A* as inverse square: $\lim_{(r\to\infty)} B_{zT} = 8/9$.

Consider now source contributions from currents. A locally uniform magnetic field may be described as: $A_L = \hat{\phi} \frac{1}{2} B_L r \sin(\theta)$, and we construct an energy term from current: $j_e \cdot A_L = \frac{1}{2} B_L e^{-r} \sin^2(\theta)$. This is easily integrated in *θ* and radially: $\int r^2 dr \sin(\theta) d\theta\, j_e \cdot A_L = 4/3 B_L$, so we can say there is a contribution to the magnetic moment from electron current, 4/3. A more subtle issue is a far-field contribution from $j_L \cdot A_e$. Given that the far electron field is $2/3 r^{-2}$, what do we make of this implied source of our ideal lab field? Under realistic assumptions we will see a distinct residue here, depending on field construction. A simple assumption of a lab current loop would yield gradients in magnetic field above and below the loop plane. We can consider a Helmholtz coil arrangement as the simplest way of eliminating first and second derivatives of the field in a region sufficiently large that far-field approximation is good. There is a problem here of approximating the "ideal lab field", becuase fundamentally circular sources cannot produce this! The third consideration of a solenoid source offers the most uniform far-field so we can take a sufficiently large dimension that integrals are accurate. The reader may confirm that a single loop source yields interaction response of 4/3; Helmholtz coils give 0.93; and the solenoid yields 2/3. It is gratifying to see this convergence, so I take the latter value: $\int d^3V\, j_L \cdot A_e = 2/3 B_L$.

Adding together the three contributions:

$$\mu = 8/9 + 4/3 + 2/3 = 2.89$$

All quantities in this discussion are without the final 2π from the Φ integration.

PART V: The Bohr Magneton

Consideration of quantized interactions gave us the representation of the Bohr magneton, or electron magnetic moment, as: $\mu_e = \hbar e / 2m_e c$. One of the challenges of field construction is to produce this product by separately integrating, for the electron model, the angular momentum, charge, and mass totals. Construct the former:

$$J_z = \int d^3V\, \hat{z} \cdot r \times \rho A_\phi \quad ,$$

as a source term integration of linear momentum density ρA at a radius, projected into the z-axis. This is detailed as:

$$J_Z = \int r^2 dr sin(\theta) d\theta\, d\phi\, sin(\theta) e^{-r} A_\phi$$

$$J_Z = 4/3 \int dr r^2 e^{-r} A \quad .$$

Evaluating this integral requires some subtlety in analyzing residues at the origin, as all the poles in r or lnr cancel, but contributions from γ and $ln(2)$ must not be neglected. The result is:

$$J_z = 4/3[1/8 - 4/3 + 2(\ln 2 - 2\gamma)] = 3.27$$

and is equated to $\hbar/2$.

Next we figure total charge, easily available as: $e = \int d^3V \rho = 2$, after *r and θ* integrations. Finally and more difficult, what is mass? We must get total electric and also magnetic energies, separately. It is not given that they are, here, equal; this is not an E&M wave. We trust that for any electric sources, the integral of field-squared equals that of *ρU*; similarly for magnetic energy, so that we may say for totals,

$$mc^2 = \int d^3V(\rho U + j \cdot A_\phi) \quad .$$

The first integral is easily accomplished and is equal to unity. The second reduces to: $4/3 \int dr(r) e^{-r} A = 4/3[-5/6 + 4/3(\ln 2 - \gamma)] = 1.15$, and we see magnetic energy is slightly larger than electric, by about 15%! Therefore total mass-energy is: $mc^2 = 2.15$.

Now we are in a position to multiply totals in the Bohr magneton:

$$\hbar e/2mc = 3.27 \times 2/2.15 = 3.04 \quad .$$

Comparing this to the first determination of 2.89, we see a 5% disparity. What of this? Had the answers come out too closely it would have been embarrassing because we know that intensities toward the center demand a relativistic solution which can be expected to compress the inner region. I think it reasonable to guess that the angular momentum integral will come down somewhat as it is most strongly dependent on radial spreading. The results we have here are sensible.