ELECTRON FIELD ANGULAR MOMENTUM, SOURCES AND FIELDS 1/13/11

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One may calculate total energies in an inhomogeneous EM system by either integrating the field terms, or the source terms, and results should match. In my circular current electron one may ask if this holds true for angular momentum (AM). The model is centered at the origin, so we look at the field density of linear momentum, then take the vector cross-product with radius.

Given a charge density the source term of momentum is: $P_s = \rho A$.,

while the field term uses the Poynting vector: $$P_f = \frac{E \times B}{c^2}$$

At any location the contribution to AM is: $S = r \times P$.

Let us take the difference of these two terms and integrate over all space, to see if they are indeed equal totals. I shall leave out the polar angle dependences in $\sin\theta$, as they add nothing to the problem. Also. Set c=1:

$$\Delta = \int_V d^3V \; r \times [\rho A - E \times B] \; .$$

Substituting for charge and for *B*, $\Delta = \int_V d^3V \, r \times [(\nabla \cdot E) A - E \times (\nabla \times A)]$.

I shall take advantage of the simple orientations of the fields, given radial electric field, and azimuthal currents and *A*-field. The magnetic field, as the curl of A_ϕ , has components in both $\hat{r}, \hat{\theta}$, so the cross-product with '*E*' selects only the latter. Thus both terms are in $\hat{\phi}$ and the last cross-product puts it all into vector sense $-\hat{\theta}$.

By expressing these orthogonal relations we can transform the problem into one of divergence integration. Substituting for both differential operators:

$$\Delta = \int_V d^3V (-\hat{\theta}) r \, [r^{-2} A \frac{d}{dr}(r^2 E) + E \frac{1}{r} \frac{d}{dr}(rA)] \; .$$

Expand the first term: $$\Delta = -\hat{\theta} \int_V d^3V \, \{ \frac{A}{r}(2\mathrm{rE} + r^2 \frac{dE}{dr}) + E \frac{d}{dr}(rA) \} \; .$$

Clear up the form: $\Delta = -\hat{\theta}\int_V d^3V \{2\mathrm{EA} + rA\frac{dE}{dr} + EA + r\,E\frac{dA}{dr}\}$,

and finally: $\Delta = -\hat{\theta}\int_V d^3V \{r\frac{d(EA)}{dr} + 3\mathrm{EA}\}$.

The trick needed here is writing a total divergence form. We create a radial vector form: $W \equiv \hat{r}\, rEA$, and take its divergence:

$$\nabla\cdot W = \frac{1}{r^2}\frac{d}{dr}(r^3 EA) = 3\mathrm{EA} + r\frac{d(EA)}{dr} \quad .$$

Lo and behold our job is finished, since we have established the integral as that of a total divergence. Gauss' Law enables us to analyze this at far radius, where we know both E and A go as inverse square of radius. The area of the far sphere goes as r^2 , so the quantity $r^3 EA$. becomes negligibly zero, and the problem is solved.