FIELD MOMENTUM and INERTIAL M ASS 2/25/18

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ABSTRACT: Electron near-fields in Kerr geometry may be integrated for field momentum, given motion of the particle.

INTRODUCTION: In Richard Feynman's second volume of Lectures on Physics, he dedicates all of chapter 28 [1] to discussing inertial mass, and field momentum. In flatspace, the calcs are of course simple. Two factors of sinθ arise with the same integral expression we constructed for field energy density, namely the square of electric field, so the result is 2/3 of the former energy total integral. It should be ½, and this method betrays a problem in our simple former theory. With Kerr's metric solution [2] applied in particle near-fields, we might expect different results, as angular symmetry is strongly altered. Plotting lines of constant radius, or polar angle, in Kerr coordinates, yields the ellipses of *r*, and "scarab" lines perpendicular, for constant θ. Here we get expressions for electric field components in both directions in the near-field. The z-coordinate is in common with Cartesian maps, so we seek an expression for lines of constant θ to understand the local rotation of the Kerr basis with respect to our Cartesian 'picture'. With this in hand, we may integrate z-components of the momentum field. We can see the radial field lines of the two systems converge in the far-field, and also that Kerr radii approach the inner energy integration limit vertically, mostly. This is certainly so in the "latitude midrange" where the angular field component is significant.

I do not see the DKS [3] fields to be useful; the complexified inverse radial form as potential function, does not yield good field energy totals. Thus I offer analysis with my proposed near-fields. Results are highly provocative, and average the same as integrated E-squared energy. In a separate paper I show details of integrations yielding bad results with DKS fields.

We sense problems in accounting for field components in the 'equatorial' case. First, tho, if we are looking down the SPIN-axis, integration over φ is trivially 2π. To correctly write an *x*-component of field momentum, we will consider variation around the circle in φ. Consider first the 'axial' case, of momentum for an electron moving in *z*. In the Kerr geometry, we deal with both radial field components, and also in θ. Expressions are in Kerr's coordinates, as are the integrations we work with, $E_r \text{ and } E_\theta$, so I derived an expression for local coordinate system rotation, but rather than take a path of rigorous math expressions, let first try to look at the physics, keeping in mind the two very different coordinate systems here. Consider coordinate lines of constant θ, at small radius. The locus of r=0, in a CARTESIAN MAP, is the disk, *z*=0, for radii in the *x*-*y* plane ≤ *a,* the characteristic angular momentum radius of Kerr. Except for right near the edge of this disk, these radial 'spokes' exit upwards or down, vertically. Then out a few radii, they have rotated to the usual far-field. Let us call the angle of the Kerr reference frame 'β', so in the far-field β=θ. Using $c \equiv \cos\theta$ one can show:

$$\sin^2\beta = \frac{\sin^2\theta}{r^2+1} \qquad \text{and} \qquad \cos^2\beta = \frac{r^2+c^2}{r^2+1} \quad .$$

Since the integral always has sinθ dθ = -d[cosθ], this is very useful and shows the symmetric behavior of the denominator polynomial in <*r,c*>*, except* for the important distinction of r_e not going to zero. Outside the Kerr ring, we may consider the “equatorial plane” of z=0. The angle here presented as 'beta' is the relative rotation of the local Kerr coordinate system, with respect to the Cartesian map as we draw. The Kerr is a locally orthogonal system. Now, step back and consider what we need.

The magnetic field generated as the particle moves along at a low velocity (non-relativistically) describes rings about the axis of motion. We are not here working with the intrinsic particle B-field, rather with a net charge in motion. What is not at all clear is the very near-field. Since momentum density in a field is *E x B,* we analyze for the electric components orthogonal to *v,* first in the polar case: $P_z = v[\sin^2\beta E_r^2 + \cos^2\beta E_\theta^2]$.

When an electric field has been realized as gradient of a potential, it is a covariant vector. Divergence is defined only on a contravariant field. When we square the field component: , we actually mean:

$\tau = E_j E^j$ or starting with a field squared, $\tau = |E_k|^2 g^{kk}$ (no summation). This is an invariant form for energy density, highly desirable. We need use the appropriate differential volume element. The Kerr metric is diagonal except for the negligible terms in the metric. Here we are appreciating how much smaller the Schwarzschild mass radius is, compared with the Kerr AM radius for Planck's constant in an electron !! Having an invariant form for energy density means that in whatever coordinate system we work, we take the differential volume element to be:

$d^3 V = \sqrt{-D}\, dx^1 dx^2 dx^3$; $g_{11} = \dfrac{r^2+c^2}{r^2+1}$ and

$g_{22} = r^2+c^2$

for the Kerr system, and there the square root of the determinant is:

$$\sqrt{-D} = (r^2+c^2)\sin\theta$$

We want to know now the inertial mass calculated in the orthogonal sense, for motion in the *z*=0 plane, say in *x.* In the former classical calculations, we see the square of the sine, which yields integrals of 2/3. This simple radial field will calc the same, regardless of angle of motion, of course, simply because velocity is the only preferred angle in the model. Cosine-squared yields the same as sine-squared when integrating by d(θ) but here we have d(cos) and 1/3. Thus we look at Kerr near-fields which have information, dependence on polar θ. Now we must admit distinctions for an equatorial case with velocity along, say, the x-axis, and look at integrating by φ. In the polar case this gives trivially, 2π. Not so in the sideways case, and analysis shows the following expressions to be useful:

$$[\sin^2\phi + \cos^2\beta\cos^2\phi]|E_r|^2 \quad + \quad [\sin^2\phi + \sin^2\beta\cos^2\phi]|E_\theta|^2 = \Pi_x \; ,$$

an unusual expression, but it shows the different vector senses if, say, we plot going up to the pole, Contributing components of E field are those at right angles to the motion, and we look separately in the Φ=0 plane, or the Φ=π/2 plane. Each term integrates to ½ of its former value of 2π, since there is no other φ-dependence.

I offer some basic hints as to analysis on the polynomials in r^2+c^2 in the denominator. Whenever the numerator has either r^2 or c^2, integration by parts clears one order from the denominator along with the numerator term. Always a factor of ½ is introduced, from " $c\,\mathrm{d}c$ ". Only when the numerator is cleared is another factor of $\frac{1}{2r^2}$ drawn out of the polynomial under integration by *c.* Let us examine them for energy totals.

FIELD ENERGY: $E^r = \frac{1}{r^2+c^2}[1+\frac{\alpha}{r}P(\theta)]$, and $E^\theta = \frac{\alpha}{r^2(r^2+c^2)}\sin\theta\cos\theta$

where $P(\theta)$ is the second-order Legendre polynomial, equal to $3\cos^2\theta-1$. Squaring the field, the integrand along with the

$$d^3V = (r^2+c^2)\sin\theta\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi \;=-\; (r^2+c^2)\,\mathrm{d}r\,\mathrm{d}c\,\mathrm{d}\phi$$

and the metric term appropriate to each, is:

$$\tau^r = [1+\frac{\alpha}{r}P(\theta)]^2 \qquad \text{and} \qquad \tau^\theta = \frac{\alpha^2}{r^4}\sin^2\theta\cos^2\theta \;.$$

A knowledgeable reader can see the wisdom of my field constructions, here.

In fact the radial component contributes very little near-field energy. Look at the

s econd term: $\int_{r_e}^{\infty} \mathrm{d}r \int_0^1 \mathrm{d}c\, \alpha^2 r^{-4} s^2 c^2 dc.$. Substitute for the sine, *s.*

$$\int_{r_e}^{\infty} \mathrm{d}r \int_0^1 \mathrm{d}c\, \alpha^2 r^{-4} (c^2 - c^4) \quad .$$

The result of having the terms in c^n is a simple difference of fractions, so this

is easily analyzed to: $1/3\, r^{-3}(1/3 - 1/5) = \frac{2\alpha^2}{45} r_e^{-3}$.

Nominally the inner radius is taken as the classical energy radius. Thus the number I have written before, 22.5. This inversely multiplies the fine structure constant as inner energy radius.

POLAR INERTIA: We want to know now the inertial mass calculated for motion on the axis of spin. In the former classical calculations, we see the square of the sine, which yields integrals of 2/3. This simple radial field will calc the same, regardless of angle of the velocity vector, of course, simply because velocity is the only preferred angle in the model. Cosine-squared yields the same answer when integrating by d(θ) but here we have d(cos) and 1/3. We must further analyze geometry in the z=0 plane; now there is need to distinguish the equatorial case, integrating by Φ*.* In the polar case this gives trivially, 2π. This will not be so in the sideways case, and we shall deal with this. it shows the different vector senses if, say, we plot going up to the pole, in the Φ= 0 plane, or the Φ=π/2 plane, given this electron being trundled along in the x-direction. One can see the classical result by realizing there is no θ-field.

Both integrations around the circle yield π, or ½ the previous result of 2π. Thus we recover ½ [1+ 1/3] or 2/3 as per Feynman and have isotropy in the simple flatspace case.

Let us consider now Kerr geometry, and start with polar momentum, which is to say motion of the electron along its spin axis. Rather than figuring components in the Kerr system, consider the plotted fields. The plots [3] are in Cartesian coordinates, and show the source disk edge on. If we think in Cartesian form, the very near-field is only near the x-axis, with a polar angle near 90 degrees.

However, our fields are written in Kerr coordinates, and we need to complete integrations in this system, so we must deal with this. We will throw out all terms in radial field, since with the Albers fields, only the tangential field is strong near the source. Now we must be careful with field expressions, and here is where we could use our angle, β. What is the physics we seek ?

Momentum density in the field is the vector cross-product E x B. The magnetic part is just the 'blob of charge' making a circular field by moving. In this polar case, we are in for a surprise. The relevant E-field lays horizontally and so for a component perpendicular to motion, we need no projections. Squaring the field, it is in either system, of orthogonal components, so things are not complicated and for near-field calcs, only terms in E_θ matter. Thinking now in Kerr coordinates, there is a very different representation of θ, and it changes with *x* near the disk source. From here we cease writing notations for Kerr coordinates, assuming we are in Kerr space. (For physics near the disk, there is close equality, so we could use $\theta=\beta$.) As before the denominator cancels under volume integrations. This is just our field energy integrand, and we need to see a factor of ½ compared to total field energy! Whence this factor if we have no angle projections?

We are speaking of very near-field, and must consider the physics of the source. According to Alexander Burinskii [4] it is a “very thin layer” and is a boundary of the inner false vacuum. The Albers fields demand a smaller inner energy radius, however, than his theory states; he uses the fine structure constant. I use his source physics, however, as a “superconducting layer”. Given this, in the very near-field the electric field is as already stated; we may however infer a factor of ½ by virtue of the back, or lower half of the charge has no effect right here, as a magnetic source. We may assign the factor of ½ to the magnetic field strength, in this case of polar motion. This is a very near-field which will blend with a far field showing the complete source. We are not interested in physics here, however.

EQUATORIAL INERTIA: Now let us construct the equatorial inertial result. We can see the logic of the form, on page 4 above.

We write the Cartesian form first and made decisions about the angles so here:

$$P_x^2 = \cos^2\phi\,(E_r^2\cos^2\theta + E_\theta^2\sin^2\theta) + \sin^2\phi\,(E_r^2) \quad .$$

There are two terms of interest and again we take, here, $\sin\theta = 1$ but we cannot ignore the implied presence in the last term also ! We should write:

$$P_x^2 = 1/2\,E_\theta^2 + 1/2\,E_r^2$$

Field components must be transformed into Kerr space, so the rotation in the middle of page 6 is used, with angle β. On the equation's LHS are Cartesian coords, and on the right are Kerr coords:

$$E_r^2 \rightarrow E_r^2\cos^2\beta + E_\theta^2\sin^2\beta \quad \text{and} \quad E_\theta^2 \rightarrow E_\theta^2\cos^2\beta + E_r^2\sin^2\beta \quad .$$

We see that $$P_x^2 = 1/2[E_\theta \cos^2\beta + E_\theta^2 \sin^2\beta] \quad ,$$

and this is a nice surprise, having dependence cancel, and we are left with exactly thalf he result as for the field energy calcs. Have a beer !!! We are free to write the integrand as in the first case.

CONCLUSIONS: Relative values of field energy, and then polar and also equatorial inertia, of these fields are all equal at 7/105. We are comparing integral results at the same intermediate stage. To complete process, there is integration over *r* of the term in inverse fourth power, so this contributes a 1/3. We have summed over a hemisphere. (Near the ring edge the flattened ellipse does slant, but this is at very small values of the cosine of θ, which appears in the numerator of the term. As stated, this field component is strong at "mid latitudes" but dies off at the pole, and also at the ring edge.)

It is clear one needs to work in the oblate Kerr coordinate system, and also with the Albers electric fields. We have answer to Feynman's challenge.

FOOTNOTES: